x	2	5	7	8
$f(x)$	.2	.1	.5	.2

Table 1: Probabilities associated with values the random variable X can take on.

1. Let the random variable X have the following probability density function, as displayed by Table 1.

   (a) Provide `R` code to sample $N = 1001$ random variables from this un-named distribution, and store it into a variable named `x`.

   (b) Provide `R` code to approximate $P(X = 2)$ based on `x`.

   (c) Give an example of a number you might get from your answer above.

   (d) What is the expected value of X, $\mathbb{E}(X)$?

   (e) Provide `R` code to approximate $\mathbb{E}(X)$ based on `x`.

   (f) What is the standard deviation of X, $\mathbb{D}(X)$?

   (g) Provide `R` code to approximate $\mathbb{D}(X)$ based on `x`.

2. Assume the random variable $X \sim F$, for some unknown distribution function defined on the support $x \in [0, \infty)$. How can we think about/imagine/operationalize the meaning of the $P(X > 2)$? Explain using whatever tools you like, pseudo R code, math, or English.

3. Rayleigh distribution has probability density function

$$f(x|\sigma) = \frac{x}{\sigma^2} \exp\left(-x^2/(2\sigma^2)\right)$$

   and has expected value $\mathbb{E}(X) = \sigma\sqrt{\frac{\pi}{2}}$.

(a) Find the maximum likelihood estimator for σ, call it $\hat{\sigma}$.

(b) Assume you have data (plural) stored in the variable `x`. For full credit, use your work above to provide an estimate for the expected value. For partial credit, provide an alternative estimate of the expected value, which does not require the calculus above.