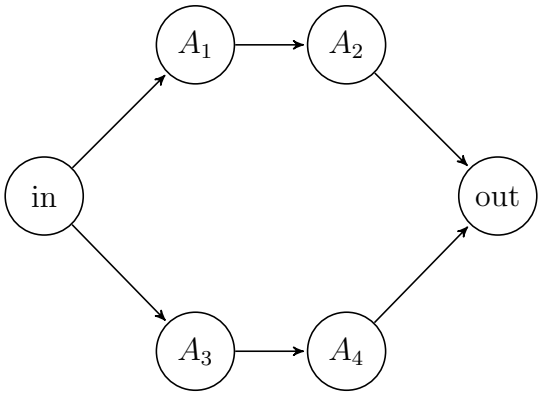
1. Let $A = \{(x, y) : x + y < 1, x, y \in \mathbb{R}\}$ and $B = \{(x, y) : x > 0, y < 0, x, y \in \mathbb{R}\}$. Sketch the region in the xy-plane corresponding to $A^c \cap B$.

2. If $P(A|B) < P(A)$, show that $P(B|A) < P(B)$.

3. Consider the following system, where the components A_1, A_2, A_3, A_4 are (mutually) independent, each with probability of success $P(A_i) = p, i = 1, 2, 3, 4$. What is the probability the system is successful?



4. A liquor store owner is willing to cash personal checks for amounts up to \$50, but she has become wary of customers who wear sunglasses. Fifty percent of checks written by persons wearing sunglasses bounce. In contrast, 98% of the checks written by persons not wearing sunglasses clear the bank. She estimates that 10% of her customers wear sunglasses. If the bank returns a check and marks it "insufficient funds," what is the probability it was written by someone wearing sunglasses?

5. A committee of fifty politicians is to be chosen from among our one hundred U.S. senators. If the selection is done at random, what is the probability that each state will be represented?

6. Let X be a discrete uniform random variable on the bounds $[a, b]$, namely $X \sim U(a, b)$. Show $E(X) = \frac{a+b}{2}$. The distribution function for X is

$$f(x) = \frac{1}{b - a + 1}, \quad x = a, a + 1, ..., b - 1, b.$$

7. Acme Industries typically produces three electric power generators a day; some pass the companys quality-control inspection on their first try and are ready to be shipped; others need to be retooled. The probability of a generator needing further work is 0.05. If a generator is ready to ship, the firm earns a profit of \$10,000. If it needs to be retooled, it ultimately costs the firm \$2,000. Let X be the random variable quantifying the companys daily profit. Find the company's expected daily profit.