1. (a) $P(B_1) = \frac{5*10^3}{10^6}$
   (b) $P(A_1) = \frac{78515}{10^6}$
   (c) $P(A_1|B_2) = \frac{73{,}630}{995{,}000}$
   (d) $P(B_1|A_1) = \frac{4885}{78515}$
   (e) i. The probability of testing positive given that the person does not carry the AIDS virus.
       ii. The probability of carrying the AIDS virus given that they tested positive.

5. (a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.7 + 0.5 - 0.9 = 0.3$
   (b) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.3}{0.5}$
   (c) $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{0.3}{0.7}$

7. (a) $\mathbb{S} = \{(Red, Red), (Red, White), (White, Red), (White, White)\}$
   (b) $P(RR|R) = \frac{P(RR\cap R)}{P(R)} = \frac{1/4}{3/4} = 1/3.$

12. 10 cartons, 2 sour, want sixth sour. Two ways this can happen: one sour in first five bought, or zero sour in first five bought. "And" converts to $\times$, and "or" converts to $+$. Mathematically, we write $P($ 6th sour | 1 in 5 sour $)P($ 1 in 5 sour $) + P($ 6th sour | 0 in 5 sour $)P($ 0 in 5 sour $)$.

    Consider the 1 sour in the first 5 bought case first. Of the first five to be bought, 2 are sour cartons and we want 1 (this is 1 of 5 cartons bought), and 8 are not sour cartons of which we want 4 (this completes the first 5). Hence, $P($ 1 in 5 sour $)$ is calculated as

$$\frac{\binom{2}{1}\binom{8}{4}}{\binom{10}{5}}.$$

    After the first five are bought, 4 not sour cartons remain of which we want 0, and 1 sour carton remains of which we want 1. So, $P($ 6th sour | 0 in 5 sour $)$ is calculated as

$$\frac{\binom{1}{1}\binom{4}{0}}{\binom{5}{1}}$$

    Conditional probability dictates then the probability that the 6th carton bought is sour, when one of the 2 sour cartons were bought within the first five is

$$P(\text{ 6th sour } | \text{ 1 in 5 sour })P(\text{ 1 in 5 sour }) = \frac{\binom{1}{1}\binom{4}{0}}{\binom{5}{1}} * \frac{\binom{2}{1}\binom{8}{4}}{\binom{10}{5}}$$

    The final answer is

$$\frac{\binom{1}{1}\binom{4}{0}}{\binom{5}{1}} * \frac{\binom{2}{1}\binom{8}{4}}{\binom{10}{5}} + \frac{\binom{2}{1}\binom{3}{0}}{\binom{5}{1}} * \frac{\binom{2}{0}\binom{8}{5}}{\binom{10}{5}}$$