

```

1. (a) ## mean
x <- c(5, 10, 15, 20, 25)
px <- 1/5
(m <- sum(x * px))

## [1] 15

## variance
sum((x - m)^2 * px)

## [1] 50

```

```

(b) ## mean
x <- 5
px <- 1
(m <- sum(x * px))

## [1] 5

## variance
sum((x - m)^2 * px)

## [1] 0

```

```

(c) ## mean
x <- 1:3
px <- (4 - x) / 6
(m <- sum(x * px))

## [1] 1.666667

## variance
sum((x - m)^2 * px)

## [1] 0.5555556

```

3 Know $\mathbb{E}(X + 4) = 10$ and $\mathbb{E}[(X + 4)^2] = 116$.

(a) By the shortcut formula, $\mathbb{V}(X + 4) = \mathbb{E}[(X + 4)^2] - \mathbb{E}(X + 4)^2 = 16$.

(b) Since $\mathbb{E}(X + 4) = \mathbb{E}(X) + 4 = 10$, we can solve for $\mu = \mathbb{E}(X) = 6$.

(c) $\mathbb{V}(X) = \mathbb{V}(X + 4) = 16$.

4 (a) $\mathbb{E}\left[\frac{X-\mu}{\sigma}\right] = \frac{\mathbb{E}(X)-\mu}{\sigma} = 0$.

(b) $\mathbb{E}\left[\left(\frac{X-\mu}{\sigma}\right)^2\right] = \frac{\mathbb{E}(X-\mu)^2}{\sigma^2} = 1$.