

- 2.5-1 (a) Because the moment generating function consists of only two terms, this is a Bernoulli random variable with probability of success $p = 2/3$. The PMF is therefore $f(x) = (2/3)^x(1 - 2/3)^{1-x}$. The Bernoulli has $\mathbb{E}(X) = p$ and $\mathbb{V}(X) = 2/3(1 - 2/3)$.
- (b) This form matches the moment generating function for the binomial random variable. Since moment generating functions are unique to each random variable, we have $X \sim \text{binomial}(12, .75)$. The PMF is therefore $f(x) = \binom{12}{x}(.75)^x(1 - .75)^{12-x}$. The binomial has $\mathbb{E}(X) = 12 * .75$ and $\mathbb{V}(X) = 12 * .75 * (1 - .75)$.
- 2.5-3 Given the moment generating function, the random variable must have probability mass function as laid out by the following table

x	1	2	3
$f(x)$	2/5	1/5	2/5

Hence, the mean must be

```
x <- 1:3
px <- c(2, 1, 2)/5
(m <- sum(x * px))                                # expected value

## [1] 2

sum((x - m)^2 * px)                               # variance

## [1] 0.8
```

Extra The moment generating function of the Binomial is $M(t) = [pe^t + (1 - p)]^k$. To find the variance, we need calculate $M''(0) - M'(0)^2$.

$$M'(0) = k * [p * e^0 + (1 - p)]^{k-1} * pe^0 = k * p$$

and

$$M''(0) = k*(k-1)[p*e^0 + (1-p)]^{k-2}*(pe^t)^2 + k(pe^0 + k(pe^0 + (1-p))^{k-1}*pe^0 = k(k-1)p^2 + kp$$

Putting these pieces together we find

$$\mathbb{V}(X) = k * (k - 1) * p^2 + kp - (k * p)^2 = (k^2 - k) * p^2 - k^2 * p^2 + kp = kp(1 - p)$$

```
2.6-1 ppois(5, 4) - ppois(2, 4)           # a
      ## [1] 0.5470271

      1 - ppois(2, 4)                     # b
      ## [1] 0.7618967

      ppois(3, 4)                         # c
      ## [1] 0.4334701
```

```
2.6-3 1 - ppois(10, 11)
      ## [1] 0.5401113
```

2.6-6 Assume the rate of flaws in aluminum screen is $1/100$ ft. Then in 50ft we should expect (i.e. on average) to see $50 * 1/100 = 1/2$ flaws. Hence, $\lambda = 1/2$.

```
ppois(0, 1/2)

## [1] 0.6065307

## Not in front of 25 people this makes more sense now. Sorry for the
## confusion, and thanks for the help in class.
```