1. Given that $P(A) = a$ and $P(B) = b$, show that
$$P(A|B) \geq \frac{a+b-1}{b}.$$

2. An urn contains three white chips, six black chips, and four red chips. Four chips are drawn in order and without replacement: black, red, white, red. What is the probability of drawing this sequence of chips?

3. Consider the set of families having two children. Assume that the four possible birth sequences – (both are boys), (younger child is a boy, older child is a girl), (younger child is a girl, older child is a boy), and (both are girls) – are equally likely. What is the probability that both children are boys given (conditioned on) at least one is a boy? Hint: the answer is not 1/2.

4. A man has n keys on a key ring, one of which opens the door to his apartment. Having celebrated a bit too much one evening, he returns home only to find himself unable to distinguish one key from another. Resourceful, he works out a fiendishly clever plan: He will choose a key at random and try it. If it fails to open the door, he will discard it, and choose at random one of the remaining $n - 1$ keys, and so on. Clearly, the probability that he gains entrance with the first key he selects is $1/n$. Show that the probability the door opens with the third key he tries is also $1/n$.