1. Suppose $P(A) + P(B) = 0.9, P(A|B) = 0.5$, and $P(B|A) = 0.1$. Calculate $P(A)$. We have $P(A|B) = \frac{P(A \cap B)}{P(B)} = 0.5$ and $P(B|A) = \frac{P(A \cap B)}{P(B)} = 0.1$. Using $P(B) = 0.9 - P(A)$, we can substitute this into $P(A \cap B)P(B) = 0.5$.. Folling that through we can solve for $P(A \cap B) = 6/9$. The plug this back into $P(B|A)$. In the end $P(A) = 9/60$.

2. Suppose two cards are drawn successively and without replacement from a standard deck of cards. Calculate the probability of drawing
   - two hearts $P(hearton2|hearton1)P(hearton1) = \frac{12}{51}\frac{13}{52}$
   - a diamond and then a spade $P(spade|diamond)P(diamond) = \frac{13}{51}\frac{13}{52}$
   - a heart and then a five.

     There's two ways to draw a five after a heart: a five of heart first and then a different five or a heart (not a five) and then a five. $P(five|heart)P(heart) = \frac{3}{51}\frac{1}{52} + \frac{4}{51}\frac{12}{52}$

3. Suppose that the genes for eye color for a certain male fruit fly are (red, white) and the genes for eye color for the mating female fruit fly are (red, white). Their offspring receive one gene for eye color from each parent.

   (a) Define the sample space for the genes of the eye color of the offpsring. {(rr), (rw), (wr), (ww)

   (b) Suppose each of the four genes for the eye color of the offspring are equally likely. Further, suppose red will dominate the eye color, such that only when the offspring receives (white, white) will they have white eyes. Given that the offspring's eyes are red, what is the probability it has gene (red, red)?

   $P(rr|\text{ red eyes }) = \frac{P(\text{ red eyes}\cap rr)}{P(\text{ red eyes })} = \frac{P(rr)}{P(\text{ red eyes })} = \frac{1/4}{3/4} = 1/3.$

4. In a string of 12 Christmas tree light bulbs, 3 are defective. The bulbs are selected at random and tested, one at a time, until the third defective bulb is found. Compute the probability that the third defective bulb is the
   - third bulb tested The first two bulbs must have been defective, so that the third buld tested is the third defective. $P(\text{ 3 is third def. } | \text{ 1,2 are def. })P(\text{ 1,2 are def.}) = \frac{\binom{1}{1}\binom{9}{0}}{\binom{10}{1}}\frac{\binom{3}{2}\binom{9}{0}}{\binom{12}{2}}$
   - fifth bulb tested

     The first 4 bulbs must have has 2 defective, so that the fifth buld tested is the third defective. $P(\text{ 5 is third def. } | \text{ 1-4 had 2 def. })P(\text{ 1-4 had 2 def.}) = \frac{\binom{1}{1}\binom{7}{0}}{\binom{8}{1}}\frac{\binom{3}{2}\binom{9}{2}}{\binom{12}{4}}$
   - tenth bulb tested The first 9 bulbs must have has 2 defective, so that the tenth buld tested is the third defective. $P(\text{ 10 is third def. } | \text{ 1-9 had 2 def. })P(\text{ 1-9 had 2 def.}) = \frac{\binom{1}{1}\binom{2}{0}}{\binom{3}{1}}\frac{\binom{3}{2}\binom{9}{7}}{\binom{12}{9}}$