

1. Let $f(x) = \frac{6}{\pi^2 x^2}$ for $x = 1, 2, 3, \dots$. Show that $f(x)$ is a probability mass function. Show that $\mathbb{E}(X)$ does not exist.
2. Let X be a random variable with support $\{1, 2, 3, 5, 15, 25, 50\}$, for which each point has probability $1/7$. Argue that $c = 5$ is the value that minimizes $h(c) = \mathbb{E}(|X - c|)$. Compare with the value of b that minimizes $g(b) = \mathbb{E}[(X - b)^2]$.
3. Let X be a discrete uniform random variable on the bounds $[a, b]$ for $0 < a < b$, namely $X \sim U(a, b)$. Show $\mathbb{E}(X) = \frac{a+b}{2}$. The distribution function for X is

$$f(x) = \frac{1}{b - a + 1}, \quad x = a, a + 1, \dots, b - 1, b.$$

Hint: What's the formula for the sum of the first n positive integers?

4. A fair coin is flipped until a head appears. You will be given $(1/2)^k$ dollars if the first head appears on the k th flip. What is the expected payout? Hint: first find the probability mass function, then determine the payout function and use it to calculate the general expected value.