

1. Suppose $X \sim \text{binomial}(K = 10, p = \frac{2}{5})$. What is the expected value of $3X - 4$?

$$\mathbb{E}(3X - 4) = 3\mathbb{E}(X) - 4 = 3 * 10 * \frac{2}{5} - 4 = 8$$

2. Suppose $U \sim U(0, 1)$. Let $Y = (b - a)U + a$. Calculate the expected value of Y .

$$\mathbb{E}((b - a)U + a) = (b - a)\frac{1}{2} + a = \frac{b+a}{2}.$$

3. If Y denote a temperature recorded in degrees Fahrenheit, then $\frac{5}{9}(Y - 32)$ is the corresponding temperature in degrees Celsius. Assume the standard deviation for a data set of temperatures is $15.7^\circ F$. What is the standard deviation in Celsius?

First, the variance: $\mathbb{V}(\frac{5}{9}(Y - 32)) = (\frac{5}{9})^2 \mathbb{V}(Y) = (\frac{5}{9})^2 * 15.7^2$. Since the standard deviation is the square root of the variance, we have $(\frac{5}{9}) * 15.7$, which is why we often prefer the standard deviation.

4. It is believed that approximately 65% of American under the age of 65 have private health insurance. Let X equal the number of Americans under the age of 65 in a random sample of $k = 15$ that have private health insurance.

- How is X distributed? $X \sim \text{binomial}(15, .65)$
- Give the mean, variance, and standard deviation of X . $\mathbb{E}(X) = 15 * .65, \mathbb{V}(X) = 15 * .65 * .35, sd(X) = \sqrt{15 * .65 * .35}$.
- Find the probability that exactly 10 have private health insurance. $P(X = 10) = \text{dbinom}(10, 15, 0.65) = .21$
- Find the probability that at most 10 have private health insurance. $P(X \leq 10) = \text{pbinom}(10, 15, 0.65) = .65$
- Find the probability that at least 10 have private health insurance. $P(X \geq 10) = 1 - \text{pbinom}(9, 15, 0.65) = .56$