1. Suppose $X \sim \textit{binomial}(K = 10, p = \frac{2}{5})$. Provide a random variable who's expected value is 3.

   $Y = \frac{3}{4}X$.

2. Let $X \sim$ geometric$(p)$ where $p$ denotes the probability of success of an underlying Bernoulli random variable. The geometric distribution has probability mass function $f(x) = p(1-p)^{x-1}$ for $x = 1, 2, \ldots$.

   (a) Provide an interpretation for the geometric random variable.

   Since $p$ is the probability of success, the PMF gives the probability of one success and $x - 1$ failures. The geometric random variable then records the probability that we see the first success on the $x$ trial.

   (b) Show that the moment generating function for the geometric random variable is $M(t) = \frac{pe^t}{1-(1-p)e^t}$ when $(1-p)e^t < 1$. Hint: to calculate this MGF, multiple by $1 = e^t e^{-t}$ to your advantage.

$$\mathbb{E}(e^{tX}) = \sum_{x=1}^{\infty} e^{tx} p(1-p)^{(x-1)} e^t e^{-t} = pe^t \sum_{x=1}^{\infty} [e^t(1-p)]^{(x-1)} = \frac{pe^t}{1-(1-p)e^t}$$

   (c) Using the moment generating function, show that the expected value (mean) of a geometric random variable is $p^{-1}$.

   The trickiest part is getting the derivative right:$M'(t) = pe^t[1-(1-p)e^t]^{-1} + [1-(1-p)e^t]^{-2}(1-p)pe^t$. Evaluating at $t = 0$ should get you there.

3. Suppose that the percentage of college students who engaged in binge drinking, which is defined as having five drinks per day (I definitely don't know the real definition of binge drinking) during the previous two weeks is approximately 40%. Let $X$ equal the number of students in a random sample of size 12 who binge drink.

   (a) Find the probability that $X$ is most 5. `pbinom(5, 12, .4)`

   (b) Find the probability that $X$ is least 6.`1 - pbinom(5, 12, .4)`

   (c) Find the probability that $X$ is equal 7. `dbinom(7, 12, .4)`

   (d) Give the mean, variance, and standard deviation of $X$.

   $\mathbb{E}(X) = 12 * .4$, $\mathbb{V}(X) = 12 * .4 * .6$, and the standard deviation is just the square root of $\mathbb{V}(X)$.