

**Poisson**

1. Flaws in a certain type of drapery material appear on average of 1 in 150 square feet. What is the probability that at most one flaw appears in 225 feet of drapery?
2. Suppose that 1 out of every 200 people suffer a side effect from a certain flu vaccine. If 1000 people get the flu vaccine, what is the probability that
  - (a) At most 1 person suffers a side effect,
  - (b) 4, 5, or 6 people suffer a a side effect?

**(Continuous) Uniform** The random variable  $X$  following the continuous uniform distribution on the interval  $[a, b]$ ,  $X \sim U(a, b)$ , has probability density function

$$f(x) = \frac{1}{b - a}.$$

1. Find the cumulative distribution function,  $F(x)$ .
2. Find the mean and variance of  $X$ . You decide which will be easier, direct calculations or using moment generating functions.

**Exponential** The random variable  $X$  following the exponential distribution on the interval  $[0, \infty)$ ,  $X \sim \text{Exponential}(\theta)$ , has probability density function

$$f(x) = \frac{1}{\theta} e^{-x/\theta}.$$

1. Find the cumulative distribution function,  $F(x)$ .
2. Calculate the median, the 50<sup>th</sup> percentile, of  $X$ .
3. Suppose that *fone* cell phones have a mean life time of 1000 days. Using the R function `pexp(x, 1/θ)`, calculate the probability a randomly chosen *fone* lasts longer than 1095 days.

**Gamma** The random variable  $X$  following the gamma distribution (not function) on the interval  $[0, \infty)$ ,  $X \sim \text{Gamma}(\alpha, \theta)$ , has probability density function

$$f(x) = (\Gamma(\alpha)\theta^\alpha)^{-1} x^{\alpha-1} e^{-x/\theta},$$

where  $\Gamma(\cdot)$  is the gamma function, and moment generating function

$$M(t) = (1 - t\theta)^{-\alpha}$$

1. Calculate the mean and variance of  $X$ .
2. Suppose the number of customers per hour arriving at a store follows a Poisson process with mean 30. Using the R function `pgamma(x, α, 1/θ)`, what is the probability that the shopkeeper will wait more than 5 minutes before the second customer arrives?