Poisson

1. Flaws in a certain type of drapery material appear on average of 1 in 150 square feet. What is the probability that at most one flaw appears in 225 feet of drapery?

```
ppois(1, 225 / 150)

## [1] 0.5578254
```

2. Suppose that 1 out of every 200 people suffer a side effect from a certain flu vaccine. If 1000 people get the flu vaccine, what is the probability that
   (a) At most 1 person suffers a side effect,
   (b) 4, 5, or 6 people suffer a a side effect?

```
ppois(1, 1000 / 200)                    # a

## [1] 0.04042768

sum(dpois(4:6, 1000 / 200))             # b

## [1] 0.4971575
```

(Continuous) Uniform The random variable X following the continuous uniform distribution on the interval $[a, b]$, $X \sim U(a, b)$, has probability density function

$$f(x) = \frac{1}{b-a}.$$

1. Find the cumulative distribution function, $F(x)$.
   $F(x) = \int_a^x \frac{t}{b-a} dt = \frac{x-a}{b-a}$
2. Find the mean and variance of X. You decide which will be easier, direct caclulations or using moment generating functions.
   Mean: $\mathbb{E}(X) = \int_a^b \frac{x}{(b-a)} dx = \frac{b^2-a^2}{2(b-a)} = \frac{b+a}{2}$
   Variance: $\mathbb{V}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$. So we should first find $\mathbb{E}(X^2)$.
   $\mathbb{E}(X^2) = \int_a^b \frac{x^2}{b-a} dx = \frac{x^3}{3(b-a)} = \frac{b^3-a^3}{3(b-a)}$
   $\mathbb{V}(X) = \frac{(b-a)^2}{12}$

Exponential The random variable X following the exponential distribution on the interval $[0, \infty)$, $X \sim \text{Exponential}(\theta)$, has probability density function

$$f(x) = \frac{1}{\theta} e^{-x/\theta}.$$

1. Find the cumulative distribution function, $F(x)$. $F(x) = \int_0^x \frac{1}{\theta} e^{-t/\theta} dt = -e^{-t/\theta} = 1 - e^{-x/\theta}$.

2. Calculate the median, the 50^{th} percentile, of X. Let's say $\theta = 1$. We are to find x such that $F(x) = 0.5$

```
-log(0.5)

## [1] 0.6931472

## or
qexp(0.5)

## [1] 0.6931472
```

3. Suppose that $fone$ cell phones have a mean life time of 1000 days. Using the `R` function `pexp(x, 1/θ)`, calculate the probability a randomly chosen $fone$ lasts longer than 1095 days.

```
1 - pexp(1095, 1/1000)

## [1] 0.3345396
```

Gamma The random variable X following the gamma distribution (not function) on the interval $[0, \infty)$, $X \sim \text{Gamma}(\alpha, \theta)$, has probability density function

$$f(x) = (\Gamma(\alpha)\theta^\alpha)^{-1} x^{\alpha-1} e^{-x/\theta},$$

where $\Gamma(\cdot)$ is the gamma function, and moment generating function

$$M(t) = (1 - t\theta)^{-\alpha}$$

1. Calculuate the mean and variance of X. $\mathbb{E}(X) = M'(0) = \alpha\theta(1 - 0 * \theta)^{-(\alpha+1)} = \alpha\theta$. $\mathbb{V}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$. $\mathbb{E}(X^2) = M''(0) = (\alpha+1)\alpha\theta^2(1 - \theta * 0)^{-(\alpha+2)} = (\alpha+1)\alpha\theta^2$. $\mathbb{V}(X) = \alpha\theta^2$.

2. Suppose the number of customers per hour arriving at a store follows a Poisson process with mean 30. Using the `R` function `pgamma(x, α, 1/θ)`, what is the probability that the shopkeeper will wait more than 5 minutes before the second customer arrives?

```
1 - pgamma(5, 2, 30 / 60)

## [1] 0.2872975
```