

1. Define the joint probability mass function of X, Y to be $f(x, y) = 1/5$ on the set of points $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$. Calculate the following
 - (a) the marginal PMF of X ,
 - (b) the marginal PMF of Y ,
 - (c) $\mathbb{E}(XY)$
 - (d) $\mathbb{E}(X)$
 - (e) $\mathbb{E}(Y)$
 - (f) $\text{Cov}(X, Y)$
2. If X and Y are independent random variables, what is the correlation between X and Y , namely ρ ? Explain how you know this.
3. If ρ , the correlation between X and Y , is equal to zero, are X and Y necessarily independent? Explain how you know this.
4. Let (X, Y) be a bivariate, continuous Uniform random variable defined on $0 < x < 1$ and $0 < y < 1$.
 - (a) Draw the support of the joint random variable.
 - (b) Draw and shade the area defined by $x^2 + y^2 \leq 1$, contained within the support of (X, Y) .
 - (c) Define the random variable W as

$$W = \begin{cases} 1 & \text{if } X^2 + Y^2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

W is a univariate random variable with a common name. What is the name of the distribution of W ?

- (d) What is the probability that $P(W = 1)$?
 - (e) What is the expected value of W , namely $\mathbb{E}(W)$?
 - (f) Provide a new random variable, P , as a function of W , who's expected value is π .
5. **Optional Challenge Question 1.** Let X and Y be defined such that $\mathbb{E}(X) = \mu_X$, $\mathbb{V}(X) = \sigma_X^2$ and $\mathbb{E}(Y) = \mu_Y$, $\mathbb{V}(Y) = \sigma_Y^2$. The values of (β_0, β_1) that minimize the function $K(\beta_0, \beta_1) = \mathbb{E}[(Y - (\beta_0 + \beta_1 * X))^2]$ are the values that define the least squares regression line – the line most often fit through a scatter plot of data. Find the values of (β_0, β_1) that minimize K . Show that $a = \mu_y - \mu_x b$ and $b = \text{Cov}(X, Y) / \sigma_X^2$.
 6. **Optional Challenge Question 2.** Suppose X and Y are random variables such that $\mathbb{E}(X) = \mu_X$ and $\mathbb{E}(Y) = \mu_Y$. Using the short-cut formula for the variance prove that $\mathbb{V}(aX + bY) = a^2\mathbb{V}(X) + b^2\mathbb{V}(Y) + 2ab\text{Cov}(X, Y)$, for constants a, b .