1. Define the joint PMF of $X, Y$ as

$$f(x,y) = \frac{x+y}{32}, \quad x = 1,2,\ y = 1,2,3,4.$$

   (a) Find the marginal of $X$. $f(x) = \frac{4x+10}{32}$ for $x = 1,2$.
   (b) Find the marginal of $Y$. $f(y) = \frac{2y+3}{32}$ for $y = 1,2,3,4$.
   (c) Find $P(X > Y)$. $\sum_{\{X>Y\}} f(x,y) = \frac{2+1}{32} = 3/32$
   (d) Find $P(Y = 2X)$. $\sum_{\{Y=2X\}} f(x,y) = \frac{1+2}{32} + \frac{2+4}{32} = 9/32$
   (e) Find $P(X + Y = 3)$. $\sum_{\{X+Y=3\}} f(x,y) = \frac{1+2}{32} + \frac{2+1}{32} = 6/32$
   (f) Find $P(X \leq 3 - Y)$. $\sum_{\{X \leq 3-Y\}} f(x,y) = \frac{1+2}{32} + \frac{2+1}{32} = 6/32$
   (g) Are $X$ and $Y$ independent? No, because $f(x)f(y) \neq f(x,y)$.

2. The probability that a person will die from a certain respiratory infection is 0.002. Of the next 2000 infected, find the After lots of thought, I don't care whether you want to solve this with a Binomial or a Poisson distribution. I'll show both.

   (a) probability that fewer than 5 will die,
   (b) probability that greater than 7 will die,
   (c) probability that exactly 10 will die,
   (d) expected number of infected people that might die.

```r
## Binomial
k <- 2000
p <- .002
pbinom(4, k, p)                  # a

## [1] 0.628837

1 - pbinom(7, k, p)              # b

## [1] 0.05095487

dbinom(10, k, p)                 # c

## [1] 0.005258077

k*p                              # d

## [1] 4
```

```
## Poisson
lambda <- k*p
ppois(4, lambda)                 # a

## [1] 0.6288369

1 - ppois(7, lambda)             # b

## [1] 0.05113362

dpois(10, lambda)                # c

## [1] 0.005292477

lambda                           # d

## [1] 4
```

3. Assume the random variables $X$ and $Y$ are independent with marginals $f(x) = 2x$ on $0 \leq x \leq 1$, and $f(y) = 3y^2$ on $0 \leq y \leq 1$. Find $P(Y < X)$.

   First, we need to find $f(x, y)$ since the probability of interest has two random variables in it. Because $X$ and $Y$ are independent, we can multiple the marginals together to get the joint density function.

   $f(x, y) = 6xy^2$.

   $P(Y < X) = \int_0^1 \int_0^x 6xy^2 dydx = \int_0^1 2xy^3\big|_0^x dx = \int_0^1 2x^4 dx = \frac{2}{5}x^5\big|_0^1 = \frac{2}{5}$.

4. Let $X \sim Exponential(\lambda)$. Show that

$$P(X > s + t | X > t) = P(X > s), \forall s, t \geq 0.$$

   This is called the memoryless property of random variables.

   By the definition of conditional probability, $P(X > s + t | x > t) = \frac{P(X > s+t)}{P(X > t)}$.

   To find the numerator and denominator we integrate the density function of the exponential distribution.

   $P(X > s + t) = \int_{s+t}^{\infty} \frac{1}{\theta} e^{-x/\theta} dx = -e^{-x/\theta}\big|_{s+t}^{\infty} = e^{-(s+t)/\theta}$

   Recognizing the likeness of the numerator and the denominator, we put the last two pieces together.

   $P(X > s + t | X > t) = \frac{e^{-(s+t)/\theta}}{e^{-(t)/\theta}} = e^{-s/\theta} = P(X > s)$.