

1. Define $f(x, y) = \lambda^2 e^{-\lambda(x+y)}$ on $0 \leq x < \infty$ and $0 \leq y < \infty$. Show that X and Y are independent.
2. Let X have a continuous uniform distribution $U(0, 2)$, and let the conditional distribution of $Y|x$ be $U(0, x^2)$. Hint: In the joint PDF x is defined on $\sqrt{y} \leq x \leq 2$.
 - (a) Determine the joint PDF of X and Y , $f(x, y)$.
 - (b) Calculate the marginal PDF of Y , $f(y)$.
 - (c) Find $\mathbb{E}(X|y)$.
3. Suppose that, on average, 1 person in 1000 makes a numerical error in preparing their income tax return. If 10,000 forms are selected at random and examined, find the probability that 6, 7, or 8 of the forms contain an error.
4. Suppose that X and Y have the following (joint) PDF

$$f(x, y) = \frac{2}{5}(2x + 3y)$$

on $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Find

- (a) $f(x)$.
- (b) $f(y|x)$.
- (c) $P(1/4 \leq Y \leq 3/4|x = 1/2)$.
- (d) $\mathbb{E}(Y|x)$.