1. Define $f(x, y) = \lambda^2 e^{-\lambda(x+y)}$ on $0 \le x < \infty$ and $0 \le y < \infty$. Show that X and Y are independent.

   $f(x) = \int_0^\infty \lambda^2 e^{-\lambda(x+y)} dy = -\lambda e^{-\lambda(x+y)}\big|_0^\infty = \lambda e^{-\lambda x}$

   By symmetry, $f(y)$ must be $f(y) = \lambda e^{-\lambda y}$. Hence, X and Y are independent.

2. Let X have a continuous uniform distribution $U(0, 2)$, and let the conditional distribution of $Y|x$ be $U(0, x^2)$. Hint: In the joint PDF x is defined on $\sqrt{y} \le x \le 2$.

   (a) Determine the joint PDF of X and Y, $f(x, y)$. $f(x) = 1/2$ and $f(y|x) = 1/x^2$. By conditional random variables, $f(x, y) = f(x)f(y|x) = 1/(2x^2)$.

   (b) Calculate the marginal PDF of Y, $f(y)$.

   $\int_{\sqrt{y}}^2 \frac{1}{2x^2} dx = \frac{-1}{2} x^{-1}\big|_{\sqrt{y}}^2 = \frac{-1}{4} + \frac{1}{2\sqrt{y}} = \frac{2-\sqrt{y}}{4\sqrt{y}}$

   (c) Find $\mathbb{E}(X|y)$.

   First we need to find the conditional density of $X|y$.

   $f(x|y) = \frac{f(x,y)}{f(y)} = \frac{4\sqrt{y}}{2x^2(2-\sqrt{y})}$

   $\frac{4\sqrt{y}}{2-\sqrt{y}} = \int_{\sqrt{y}}^2 (2x)^{-1} dx = \frac{4\sqrt{y}}{2-\sqrt{y}} * \frac{1}{2} \log x\big|_{\sqrt{y}}^2 = \frac{4\sqrt{y}}{2*(2-\sqrt{y})} * (\log 2 - \log \sqrt{2})$

3. Suppose that, on average, 1 person in 1000 makes a numerical error in preparing their income tax return. If 10,000 forms are selected at random and examined, find the probability that 6, 7, or 8 of the forms contain an error.

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## I don't care if you thought Poisson here or Binomial.

## Binomial
sum(dbinom(6:8, 10000, 1/1000))

## [1] 0.2657156

## Poisson
sum(dpois(6:8, 10000/1000))

## [1] 0.2657337
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4. Suppose that X and Y have the following (joint) PDF

$$f(x, y) = \frac{2}{5}(2x + 3y)$$

   on $0 \le x \le 1$ and $0 \le y \le 1$. Find

   (a) $f(x)$.

   $f(x) = \int_0^1 \frac{2}{5}(2x + 3y)dy = \frac{2}{5}(2xy + 3y^2/2)\big|_0^1 = \frac{2}{5}(2x + 3/2)$.

(b) $f(y|x)$. $f(y|x) = \frac{2x+3y}{(2x+3/2)}$

(c) $P(1/4 \leq Y \leq 3/4|x = 1/2) = \int_{1/4}^{3/4} \frac{2}{5}(1+3y)dy = \frac{2}{5}(y + 3y^2/2)\Big|_{1/4}^{3/4} = 1/2.$

(d) $\mathbb{E}(Y|x) = \int_0^1 \frac{2xy+3y^2}{(2x+3/2)}dy = \frac{xy^2+y^3}{2x+3/2}\Big|_0^1 = \frac{x+1}{2x+3/2}.$