

1. Suppose that the random variables  $W, X, Y$ , and  $Z$  have PDF

$$f(w, x, y, z) = 16wxyz$$

on  $0 < w < 1, 0 < x < 1, 0 < y < 1, 0 < z < 1$ .

- (a) Find the marginal distribution  $f(w, x)$ .  $f(w, x) = \int_0^1 \int_0^1 16wxyz dy dz = 4wx$
- (b) Find  $\mathbb{E}(XW)$ .  $\mathbb{E}(XW) = \int_0^1 \int_0^1 4x^2w^2 dw dx = 4/9$
- (c) Find the expected value of  $X$ , namely  $\mathbb{E}(X)$ .  $\mathbb{E}(X) = \int_0^1 xf(x) dx = \int_0^1 x * 2x dx = 2/3$
- (d) Are  $X$  and  $W$  independent? Yes, since  $f(x) = 2x$  and  $f(w) = 2w$  multiply to give  $f(w, x)$ .
- (e) Find  $\text{Cov}(X, W)$ .  $\text{Cov}(X, W) = 0$  since  $X$  and  $W$  are independent.
2. Find  $P(X < 1 | y = 3/2)$  if  $X$  and  $Y$  have (joint) PDF

$$f(x, y) = xy/2$$

on  $0 \leq x \leq y \leq 2$ .

$P(X < 1 | y = 3/2) = \int_0^1 f(x|y) dx$ . So we need to find  $f(x|y)$ , which means we need to find  $f(y)$  first.

$$f(y) = \int_0^y xy/2 dx = y^3/4$$

$$f(x|y) = 2x/y^2$$

$$P(X < 1 | y = 3/2) = \int_0^1 = \frac{8}{9} \int_0^1 x dx = 4/9.$$

3. To avoid detection at customs, a traveler places 6 narcotics tablets in a bottle containing 9 vitamin pills that are similar in appearance. If the customs official selects 3 of the tablets at random for analysis, what is the probability that the narcotics are found?

```
## Binomial
k <- 3
p <- 6/15
1 - pbinom(0, k, p)      # P(X >= 1), at least one narcotic pill found

## [1] 0.784
```

4. A telephone solicitor is responsible for canvassing three suburbs. In the past, 60% of the completed calls to Belle Meade have resulted in donations, compared to 55% for Oak Hill and 35% for Antioch. Her list of telephone numbers includes one thousand households from Belle Meade, one thousand from Oak Hill, and two thousand from Antioch. Suppose that she picks a number at random from the list and places the call. What is the probability that she gets a donation?

```
pD_BM <- .6
pD_OH <- .55
pD_A <- .35

pBM <- 1/4
pOH <- 1/4
pA <- 2/4

pD_BM * pBM + pD_OH * pOH + pD_A * pA

## [1] 0.4625
```